Constraint Semantics
and the Language of Subjective Uncertainty

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The doxastic hypothesis:
Propositions do not suffice to characterize a typical agent's doxastic state. A complete inventory of the propositions to which I lend high credence would omit my belief that it might rain today.

The assertion hypothesis:
Propositions suffice to characterize the contents of assertions. For example, the content of my assertion that it might rain today is a proposition about information available to me or to my community.

Kratzer's hypothesis: a given modal has a "common kernel of meaning" whether it is used to target epistemic modality, deontic modality, circumstantial modality, or some other flavor of modality (1977, 338–342). That common kernel pertains to the relationship between the modal's prejacent and a contextually supplied body of information or set of premises.

Section 1 reconciles compositional semantics and the doxastic hypothesis.

Section 2 refines Kratzer's hypothesis, and uses it to help explain the evidential features of epistemic modals.

Section 3 argues that epistemic modals demand a hybrid of 'pure' probabilistic semantics, à la section 1, and premise semantics, à la section 2.

1. Constraint Semantics

1.1. Modeling doxastic constraints

Here I model doxastic states using *probability spaces*; $\langle W, \mathcal{F}, \mu \rangle$ such that

1. $\mathcal{F}$ is an algebra over W (i.e., $\mathcal{F}$ is a set of subsets of W, $W \in \mathcal{F}$, and $\mathcal{F}$ is closed under complementation and union);
2. μ (the *measure function* of the triple) is a function from $\mathcal{F} \to [0, 1]$;
3. $\mu(W) = 1$;
4. If M and N are disjoint elements of $\mathcal{F}$, then $\mu(M \cup N) = \mu(M) + \mu(N)$.

A constraint on doxastic states is a set of probability spaces that are admissible by the lights of that constraint.

Intuitively, the constraint that should be associated with (1) is the set of probability spaces that take the proposition that it is raining now to 0.5. The constraint that should be associated with (2) is the set of probability spaces that take the proposition that the next ball drawn will be white to 0.25 or to 0.75.

(1) There's a 50% chance that it's raining now.

(2) There's a 25% chance that the next ball drawn will be white, and a 25% chance that the next ball drawn will be red.

1.2. A fragment of English

Types:

e is a type (in particular, the type of individuals: $D_{\langle e\rangle} = \{\text{Al, Betty}\}$);
t is a type (in particular, the type of truth values: $D_{\langle t\rangle} = \{\textit{true}, \textit{false}\}$);
a is a type (in particular, the type of admissibles: $D_{\langle a\rangle} = \{$the set of $\langle W, \mathcal{F}, \mu\rangle$ triples such that W is the set of all possible worlds, and $\langle W, \mathcal{F}, \mu\rangle$ is a probability space$\}$;
if α and β are types, then $\langle\alpha, \beta\rangle$ (sometimes abbreviated '$\alpha\beta$') is a type;
nothing else is a type.

Semantic entries:

$[\![\textbf{Al}]\!]_{\langle e\rangle}$ = Al; $[\![\textbf{Betty}]\!]_{\langle e\rangle}$ = Betty

$$[\![\textbf{is/are tall}]\!]_{\langle e,\langle a,t\rangle\rangle} = \lambda e.\lambda a.\begin{cases}\textit{true} \text{ if the measure function of } a \text{ takes the proposition that } e \text{ is tall to 1;}\\ \textit{false} \text{ otherwise.}\end{cases}$$

$$[\![\textbf{is/are nice}]\!]_{\langle e,\langle a,t\rangle\rangle} = \lambda e.\lambda a.\begin{cases}\textit{true} \text{ if the measure function of } a \text{ takes the proposition that } e \text{ is nice to 1;}\\ \textit{false} \text{ otherwise.}\end{cases}$$

$$[\![\textbf{there is an } x\textbf{\% chance that}]\!]_{\langle\langle a,t\rangle,\langle a,t\rangle\rangle} =$$
$$\lambda C \in D_{\langle a,t\rangle}.\lambda a.\begin{cases}\textit{true} \text{ if } a \text{ takes } p \text{ to } \frac{x}{100}, \text{ (where } p \text{ is a proposition that every measure function of } C \text{ takes to 1);}\\ \textit{false} \text{ otherwise.}\end{cases}$$

$$[\![\textbf{and}]\!] = \lambda F \in D_{\langle a,t\rangle}.\lambda G \in D_{\langle a,t\rangle}.\lambda a.\begin{cases}\textit{true} \text{ if } F(a) = \textit{true} \text{ and } G(a) = \textit{true};\\ \textit{false} \text{ otherwise.}\end{cases}$$

1.3. Negation

We might be tempted to give the following constraint semantic entry for 'it is not the case that':

$$[\![\textbf{it is not the case that}]\!] = \lambda F \in D_{\langle a,t\rangle}.$$
$$\lambda a.\begin{cases}\textit{true} \text{ if, if } F(b) = \textit{true} \text{ and } b \text{ gives } x \text{ to a proposition, then } a \text{ gives } 1 - x \text{ to that proposition;}\\ \textit{false} \text{ otherwise.}\end{cases}$$

This entry would make the right predictions about the constraint semantic value of

(3) It is not the case that Al is tall.

But it makes badly counterintuitive predictions about many epistemically hedged sentences.

(4) There is a 5% chance that Al is tall.

(5) It is not the case that there is a 95% chance that Al is tall.

(6) There is a 50% chance that Betty is nice.

(7) It is not the case that there is a 50% chance that Betty is nice.

(8) It is not the case that Al is tall.

(9) It is not the case that there is a 100% chance that Al is tall.

This semantic entry works well: $[\![\textbf{it is not the case that}]\!] = \lambda F \in D_{\langle a,t \rangle}.\lambda a. \begin{cases} \textit{true} \text{ if } F(a) = \textit{false}; \\ \textit{false} \text{ otherwise.} \end{cases}$

1.4. Disjunction

$[\![\textbf{or}]\!] = \lambda F \in D_{\langle a,t \rangle}.\lambda G \in D_{\langle a,t \rangle}.$

$$\lambda a. \begin{cases} \textit{true} \text{ if } a \text{ gives 1 to any proposition that is the union of a proposition} \\ \text{assigned 1 by every probability space that is admissible by the lights of } F \\ \text{and a proposition assigned 1 by every probability space that is admissible by the lights of } G; \\ \textit{false} \text{ otherwise.} \end{cases}$$

This works for some simple examples. But we also need to handle examples like

(10) We're either about as likely as not to hire John, or we're about as likely as not to hire James—you know how bad I am with names.

$$[\![\textbf{or}]\!] = \lambda F \in D_{\langle a,t \rangle}.\lambda G \in D_{\langle a,t \rangle}.\lambda a. \begin{cases} \textit{true} \text{ if } F(a) = \textit{true} \text{ or } G(a) = \textit{true}; \\ \textit{false} \text{ otherwise.} \end{cases}$$

(S satisfies $\mathcal{C}$ if S treats the elements of $\mathcal{C}$ as 'possible end states,' in the sense that if S's state were to *rule out* all of the elements of $\mathcal{C}$ but one, then S's state would be the element of $\mathcal{C}$ that S does not rule out.)

1.5. Quantification

(11) "Every moment you spend with your child could be the one that really matters" (Russert 2006, xv–xvi).

(12) "Každyj priëm kokaina možet stat' poslednim."[1]
Every dose of cocaine could become the last.
'Every time you take cocaine could be your last.'

(13) Given only what we can be certain of, no one here has to be the thief.

(14) Almost every square inch of the floor might have paint on it.

The semantic value of 'almost every square inch of the floor' is type $\langle\langle e, \langle a, t\rangle\rangle, \langle a, t\rangle\rangle$. It takes an open sentence like 'λx. [might [x has paint on x]]' to the characteristic function of the set of admissibles each element of which has the following property: for each square inch of the floor that is in some set of square inches on the floor consisting of almost every such square inch, the proposition that that square inch of the floor has paint on it gets at least 'might' level credence.

1.6. Constraint semantics and the force of assertion

Propositions represent ways the world might be; constraints generally do not. The move away from truth-conditional semantics is also a move away treating assertion as a kind of representation. What do we do when we assert that φ? Too strong: In asserting that φ, a speaker advises her addressees to conform their credences to the semantic value of 'φ'. Too piecemeal: In asserting that it might be that φ, a speaker *weakly* advises her addressees to conform their credences to the semantic value of 'Might φ'.

[1] http://podrobnosti.ua/health/2010/01/13/657737.html. Thanks to Daniel Altshuler and Natalia Kondrashova for their judgments.

Better: Authority reflects range. The *authority* that a speaker claims in asserting that φ decreases with increases in the size of the *range* of credences such that 'S believes that φ' is true (holding fixed context, content of the prejacent, vagueness of expression, intonation, stakes, background conditions, and …).

1.7. Assessment

The White spies are spying on the Red spies, who are spying on the gun for hire. The gun for hire has left evidence suggesting that he is in Zurich, but one clever White spy knows that he is in London. After finding the planted evidence, one Red spy says to the others, "The gun for hire might be in Zurich," and the others respond "That's true." The clever White spy says "That's false—he's in London" to the other White spies, and explains how he knows this. (cf. Egan et al. 2005)

Authority reflects range helps explain why we have relativist-friendly judgments here: the less authority we claim when making an assertion, the more lenient the norms that govern the assertion.

2. Evidentiality

G. E. Moore (foreshadowing Karttunen 1972):

> 'You *must* have omitted to turn the light off' means: 'There's conclusive evidence that you didn't.' The evidence is: It wouldn't have been on now, if you had turned it off, for (a) nobody else has been in the room & (b) switches can't turn on by themselves. But 'you certainly didn't' doesn't = 'You *must* have omitted': we shouldn't say the latter if we *saw* you come out without turning it off: we then shouldn't have *inferred* that you didn't. (1962, 188, dating to 1941–1942)

(17) John must be here by now.

The same 'evidential' signal is often carried by other English epistemic modals.

(18) John has to be here by now.

(19) John should be here by now.

(20) John ought to be here by now.

English epistemic possibility modals have a very similar feature, visible with the help of wide-scope negation.[2]

(21) John couldn't be here by now.

(22) I don't think John could be here by now.

(23) I doubt that John could be here by now.

Following Kratzer 1976 (cf. van Fraassen 1973 and Veltman 1976), I hold that all readings of 'must,' 'have to,' 'should,' 'ought,' 'can,' 'could,' 'might,' and the like pertain to the relation between the prejacent and a set of premises. This is how and why epistemic modals carry an 'evidential' signal.

[2]See Huddleston & Pullum 2002, 186, Copley 2006, Swanson 2008, 1203–1205, and von Fintel & Gillies 2009 for evidentiality in weak necessity modals; see Swanson 2006, 56 and von Fintel & Gillies 2009 for evidentiality in possibility modals.

2.1. Premise semantics for strong necessity modals

I do not endorse the Kratzer/Veltman *implementation* of premise semantics, however (for details, see my 2009). We agree that modals are evaluated relative to a set of premises, or, equivalently, as a partial preorder over possible worlds (Lewis 1981). The premises constitute arguments. Argument A is stronger than argument B iff the premises of argument A include all the premises in argument B and more besides. If neither argument includes all the premises of the other, then they are incomparable in strength. To a first approximation, on the Kratzer/Veltman semantics (24) means that 'φ' follows from all the strongest arguments available.

(24) It must be/has to be that φ.

(25) means (to a first approximation) that some strongest argument available does not falsify 'φ':

(25) It might be that φ.

More precisely:

Definition 1. *A relation is a preorder iff it is conditionally reflexive and transitive.*

Definition 2. *A preorder $\lesssim$ totally preorders a set S iff* $\forall x \forall y\big((x \in S \wedge y \in S) \rightarrow (x \lesssim y \vee y \lesssim x)\big)$.

Definition 3. $\lesssim_i$ *(read 'is at least as good as at world i') is a partial preorder of a set* $S_{\lesssim_i}$ *of worlds such that* $S_{\lesssim_i} = \{w : w \lesssim_i i \vee i \lesssim_i w\}$.

Definition 4. $<_i$ *(read 'is better than at i') is a strict partial order such that* $\forall x \forall y\big(x <_i y \leftrightarrow (x \lesssim_i y \wedge y \not\lesssim_i x)\big)$.

PM (Partial 'Must'):
'Must C' is true at i (relative to $\lesssim_i$) iff for every world $h \in S_{\lesssim_i}$ there is some world j such that $j \lesssim_i h$ and every world k such that $k \lesssim_i j$ is a C-world. (Kratzer 1981, 298; 1991, 644)

A problem case. Suppose that John and Karen deontically value proper supersets of children strictly increasingly, and that, because they think every life is uniquely precious, they think that sets of children neither of which is a subset of the other are deontically incomparable. John and Karen know that they have an unusual condition: they will have only boys unless they have an operation that will allow them to conceive one girl but will also make them infertile. They believe that they must have at most finitely many children. The numbers in the figure below indicate which children are conceived: boys have even numbers and girls have odd numbers.

(26) It must be that a girl is conceived.

(27) It must be that the last child conceived is a girl.

(28) It is permissible that only boys are conceived.

(29) It must be that only boys are conceived.

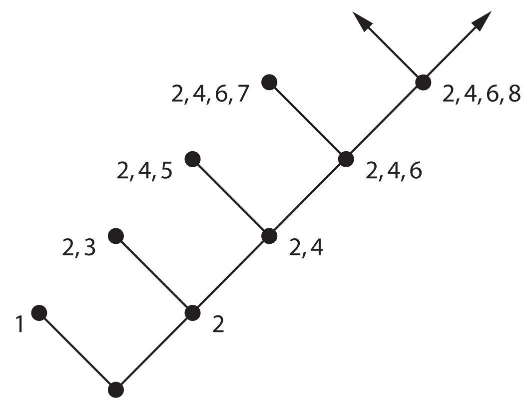
(30) If John and Karen were obligated to conceive at most n children, they wouldn't have to conceive a girl.

Definition 5. *A set S is a $\lesssim$ antichain iff* $\forall x\Big(x \in S \rightarrow \big(\exists y(x \lesssim y \vee y \lesssim x) \wedge \neg\exists z(z \in S \wedge x \lesssim z \vee z \lesssim x)\big)\Big)$.

Definition 6. *A $\lesssim$ antichain S is a maximal $\lesssim$ antichain iff no $\lesssim$ antichain properly includes S.*

Definition 7. *A set S is a $\lesssim$ chain iff $\lesssim$ totally preorders S.*

Definition 8. *A $\lesssim$ chain S is a maximal $\lesssim$ chain iff no $\lesssim$ chain properly includes S.*



Cheaper by the dozen

Kratzer's approach mishandles Cheaper by the dozen because it disregards the maximal chain consisting of worlds in which John and Karen conceive more and more boys. Another way to see the point: PM =

AM (Antichain 'Must'):

'Must $\mathcal{C}$' is true at i (relative to $\lesssim_i$) iff there is some maximal $\lesssim_i$ antichain, B, such that $\forall h \forall j\big((h \in B \wedge j \lesssim_i h) \to j \in C\big)$.

Theorem 1. *'Must $\mathcal{C}$' is true at i (relative to $\lesssim_i$) according to PM iff it is true at i (relative to $\lesssim_i$) according to AM.*

But it's possible for a maximal $\lesssim$ antichain to be disjoint from some maximal $\lesssim$ chain.

Definition 9. *A set S is a $\lesssim$ cutset iff S contains an element of each maximal $\lesssim$ chain.*[3]

CM (Cutset 'Must'):

'Must $\mathcal{C}$' is true at i (relative to $\lesssim_i$) iff there is some $\lesssim_i$ cutset, B, such that $\forall h \forall j\big((h \in B \wedge j \lesssim_i h) \to j \in C\big)$.

Lemma 1. *For each $\lesssim_i$ cutset B there is some maximal $\lesssim_i$ antichain A such that $A \subseteq B$.*

Theorem 2. *'Must $\mathcal{C}$' is true at i (relative to $\lesssim_i$) according to CM only if it is true at i (relative to $\lesssim_i$) according to AM.*

Lemma 2. *Let $t \in T$ iff $\forall k(k \lesssim_i t \to k \in C)$. If 'Must $\mathcal{C}$' is not true at i (relative to $\lesssim_i$) according to CM, then some maximal $\lesssim_i$ chain M is such that $M \cap T = \varnothing$.*

Theorem 3. *As before, let $t \in T$ iff $\forall k(k \lesssim_i t \to k \in C)$. If 'Must $\mathcal{C}$' is true at i (relative to $\lesssim_i$) according to AM and not according to CM, then there is some maximal $\lesssim_i$ chain M and some maximal $\lesssim_i$ antichain A such that $A \subseteq T$ and every element of M is $\lesssim_i$ bettered by some element of A.*

[3] For early work on cutsets see Bell & Ginsburg 1984 (where, generalized to apply to graphs as well as to preorders, they are called "transversals") and Ginsburg 1984; see also Grillet 1969. Some partially ordered sets lack minimal cutsets (Higgs 1985, Lonc & Rival 1987, and Maltby 1994) so they are not good candidates to be lower bounds.

2.2. Premise semantics for weak necessity modals

Bas van Fraassen foreshadowed premise semantics in his 1973. To a first approximation, on van Fraassen's semantics (31) means that there is *some* strongest argument available such that 'φ' follows from it.

(31) It ought to be/should be that φ.

AM/PM and CM validate agglomeration: $\mathrm{M}(\mathcal{A}) \wedge \mathrm{M}(\mathcal{B}) \vDash \mathrm{M}(\mathcal{A} \wedge \mathcal{B})$. But van Fraassen's semantics for 'ought' does not: $\mathrm{O}(\mathcal{A}) \wedge \mathrm{O}(\mathcal{B}) \nvDash \mathrm{O}(\mathcal{A} \wedge \mathcal{B})$. Kratzer's 'is a good possibility' (1991, 644) = van Fraassen's 'ought' =

OSO (Ordering Semantics 'Ought'):

'Ought $\mathcal{C}$' is true at i (relative to $\lesssim_i$) iff there is some world j such that $j \lesssim_i i$ and every world k such that $k \lesssim_i j$ is a C-world.

(32) It ought to be that the last child conceived is a girl.

(33) It ought to be that the last child conceived is a boy.

MCO (Maximal Chain 'Ought'):

'Ought $\mathcal{C}$' is true at i (relative to $\lesssim_i$) iff 'Must $\mathcal{C}$' is true at i relative to some maximal $\lesssim_i$ chain.

(32) still comes out true on MCO. But now (33) comes out true as well, because 'It must be that the last child conceived is a boy' is true relative to the maximal $\lesssim_i$ chain consisting of worlds in which only boys are conceived.

The intuitive thought is that weak necessity modals like 'ought' and 'should' abstract away from incomparability: 'Ought $\mathcal{C}$' is true iff there is some way of bracketing moral dilemmas on which 'Must $\mathcal{C}$' is true. So weak necessity modals decompose partial preorders into their constituent maximal chains and test those maximal chains against the standards associated with strong necessity modals like 'must' and 'have to.' It is plausible that it is at least part of the explanation of why (32) and (33) are consistent even though (27) and (34) aren't.

(27) It must be that the last child conceived is a girl.

(34) It must be that the last child conceived is a boy.

Recent accounts of the distinction between weak and strong necessity modals—in particular, those advocated by Kratzer 1991, Sæbø 2001, von Fintel & Iatridou 2005, Copley 2006, von Stechow et al. 2006, and von Fintel & Iatridou 2008—wrongly render (32) and (33) every bit as inconsistent as (27) and (34).

3. Weak necessity modals and the need for a hybrid theory

The quantitative aspects of the language of subjective uncertainty make pure constraint semantics look attractive. The evidential aspects of epistemic modals make premise semantics look attractive. I advocate a hybrid.

The premise semantics approach effaces important differences between strong necessity modals like 'must' and 'have to' and possibility modals like 'might,' on the one hand, and weak necessity modals like 'should' and 'ought.' It's anomalous to follow (17) or (18) with (35), but it's fine to follow (19) or (20) with (35).

(17) John must be here by now.

(19) John should be here by now.

(35) But he's not here yet.

Both weak and strong necessity modals signal that their prejacent is the conclusion of an inference, but only strong necessity epistemic modals convey that the speaker endorses any level of commitment to the prejacent. Weak necessity epistemic modals are perhaps most naturally used alongside the *denial* of the prejacent (cf. Swanson 2008, 1203–1204). This can be explained by the combination of

- epistemic strong necessity modals = CM plus a doxastic constraint wrt the prejacent,
- epistemic weak necessity modals = MCO without *any* doxastic constraint wrt the prejacent.

Surprising fact: possibility modals are stronger than weak necessity modals in an analogous way.

(36) #He left an hour ago, and there isn't any traffic. So John might be here by now, but he's not here yet.

(37) They should be here by now, but they're not.

(38) #They might be here by now, but they're not.

So (epistemic) 'ought' does not imply (epistemic) 'can.' To explain this we need a hybrid—the above plus

- epistemic possibility modals are dual to epistemic strong necessity modals, in both their premise semantic and constraint semantic aspects.

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